Polarized Radiative Transfer in Stellar Atmospheres

Gioele Janett, Oskar Steiner

Istituto Ricerche Solari Locarno

Radiation hydrodynamic simulations of stellar atmospheres frequently show shock fronts, contact discontinues, or steep gradients across which radiative transfer needs to be computed in post-processing steps. In view of applications to magnetohydrodynamic models of the solar atmosphere, we have developed a new algorithm for integrating the equation of radiative transfer for polarized light across a discontinuous atmosphere using methods of piecewise continuous reconstruction and slope limiters. In this poster, we present a comparison of results obtained with our new algorithm and with more conventional methods.

Radiative Transfer Equation and Formal Solution

The radiative transfer equation for polarized light describes the propagation of light in a absorbing and emitting atmosphere. It is usually written under the form [1]

$$\frac{d}{ds}\mathbf{I}(s) = -\mathbf{K}(s)\mathbf{I}(s) + \boldsymbol{\epsilon}(s)\,, \tag{1}$$

where $\mathbf{I} = (I, Q, U, V)$ is the Stokes vector and, in the simplest case, the absorption matrix

$$\mathbf{K} = \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix}, \tag{2}$$

and the emission vector $\boldsymbol{\epsilon} = (\epsilon_I, \epsilon_U, \epsilon_U, \epsilon_V)$ are known functions depending on the optical depth τ and on the wavelength.

Conventional Solution (DELO)

The Equation (1) can be rewritten under the form [2]

$$\frac{d}{d\tau}\mathbf{I}(\tau) = \mathbf{I}(\tau) - \mathscr{S}(\tau)\,, \tag{3}$$

where $\mathscr{S}$ is the so called effective source function.
The solution of Equation (3) in the interval (τ_k, τ_{k+1}) is given by the lambda transform

$$\mathbf{I}(\tau_k) = e^{-(\tau_{k+1}-\tau_k)}\mathbf{I}(\tau_{k+1}) + \int_{\tau_k}^{\tau_{k+1}} e^{-(\tau-\tau_k)}\mathscr{S}(\tau)d\tau\,. \tag{4}$$

In order to evaluate the integral in Equation (4), the effective source function $\mathscr{S}$ is approximated by a linear interpolation

$$\mathscr{S}(\tau) = \frac{(\tau_{k+1}-\tau)\mathscr{S}_k + (\tau-\tau_k)\mathscr{S}_{k+1}}{\tau_{k+1}-\tau_k}\,. \tag{5}$$

Here, $\mathscr{S}$ is piecewise linear and globally continuous.

Evolution Operator Solution (EVOP)

The formal solution can be also expressed in terms of the evolution operator in the interval (τ_k, τ_{k+1}) and reads

$$\mathbf{I}(\tau_k) = \mathbf{O}(\tau_k, \tau_{k+1})\mathbf{I}(\tau_{k+1}) + \int_{\tau_k}^{\tau_{k+1}} \mathbf{O}(\tau_k, \tau)\boldsymbol{\epsilon}(\tau)d\tau\,. \tag{6}$$

If the absorption matrix is assumed constant, the evolution operator can be decomposed in a linear combination of four matrices multiplied by suitable exponential factors

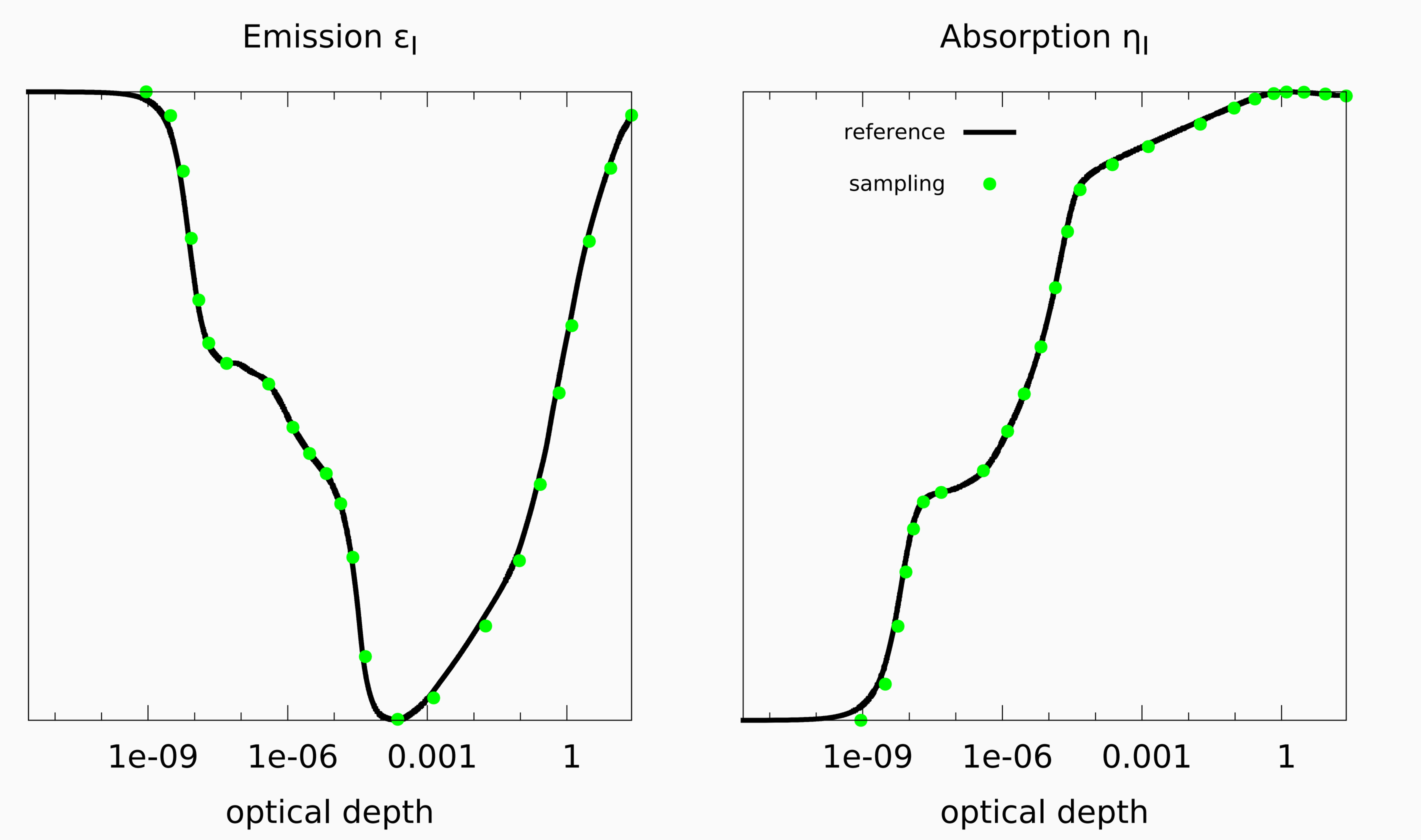
$$\mathbf{O}(\tau_k, \tau) = \sum_{i=1}^{4} e^{-(\tau-\tau_k)\lambda_i}\mathbf{N}_i\,, \tag{7}$$

where λ_i are the eigenvalues of the absorption matrix.
If also the emission vector is assumed constant, one finds

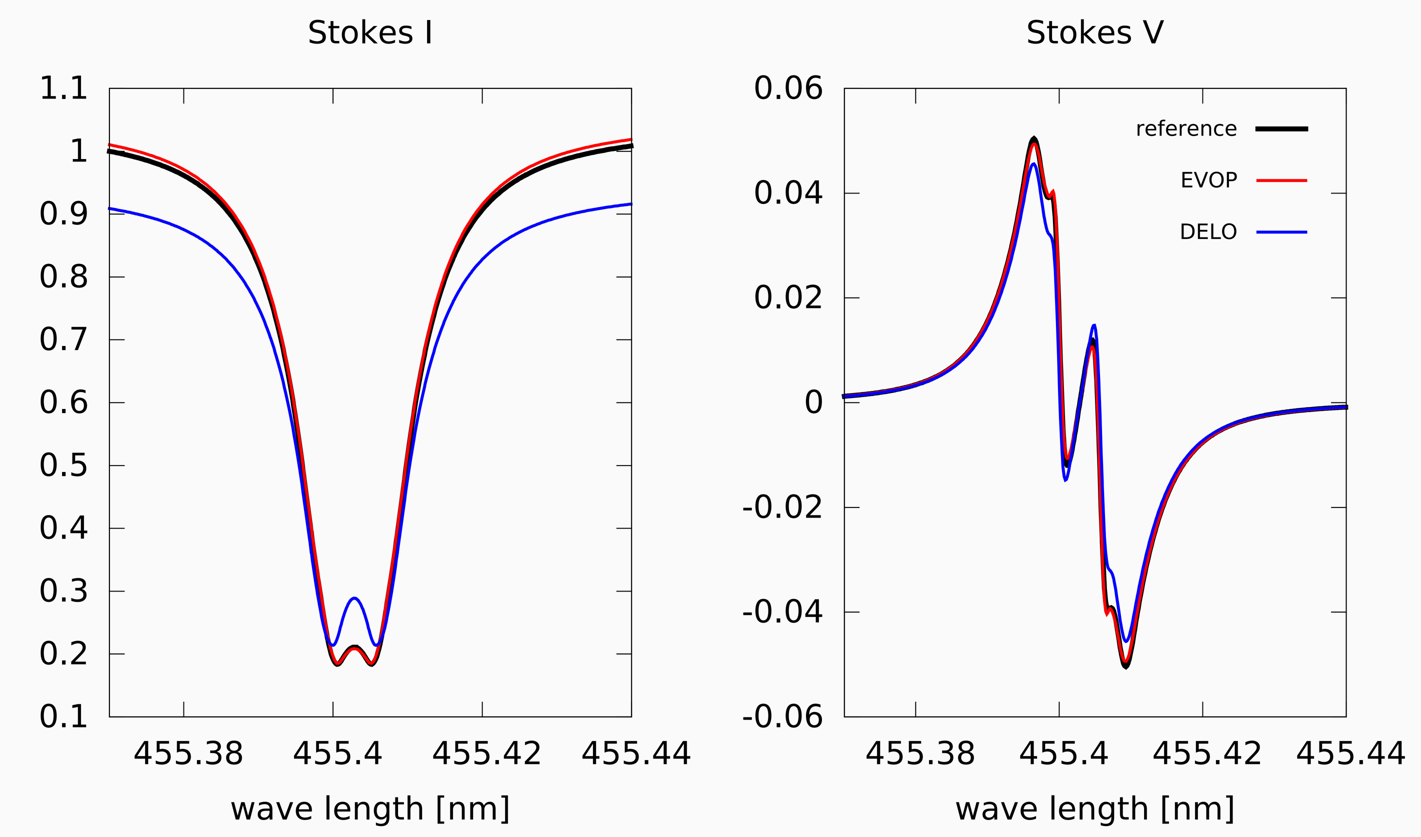
$$\mathbf{I}(\tau_k) = \mathbf{O}(\tau_k, \tau_{k+1})\mathbf{I}(\tau_{k+1}) + \left[\sum_{i=1}^{4} \frac{1-e^{-(\tau_{k+1}-\tau_k)}}{\lambda_i}\mathbf{N}_i\right]\boldsymbol{\epsilon}\,. \tag{8}$$

Here, $\boldsymbol{\epsilon}$ is piecewise constant and discontinuous at cell interfaces.
We are actually using higher order polynomials in the approximation of $\mathbf{S}$, applying slope limiters in order to avoid overshooting in case of drastic changes in the emission.

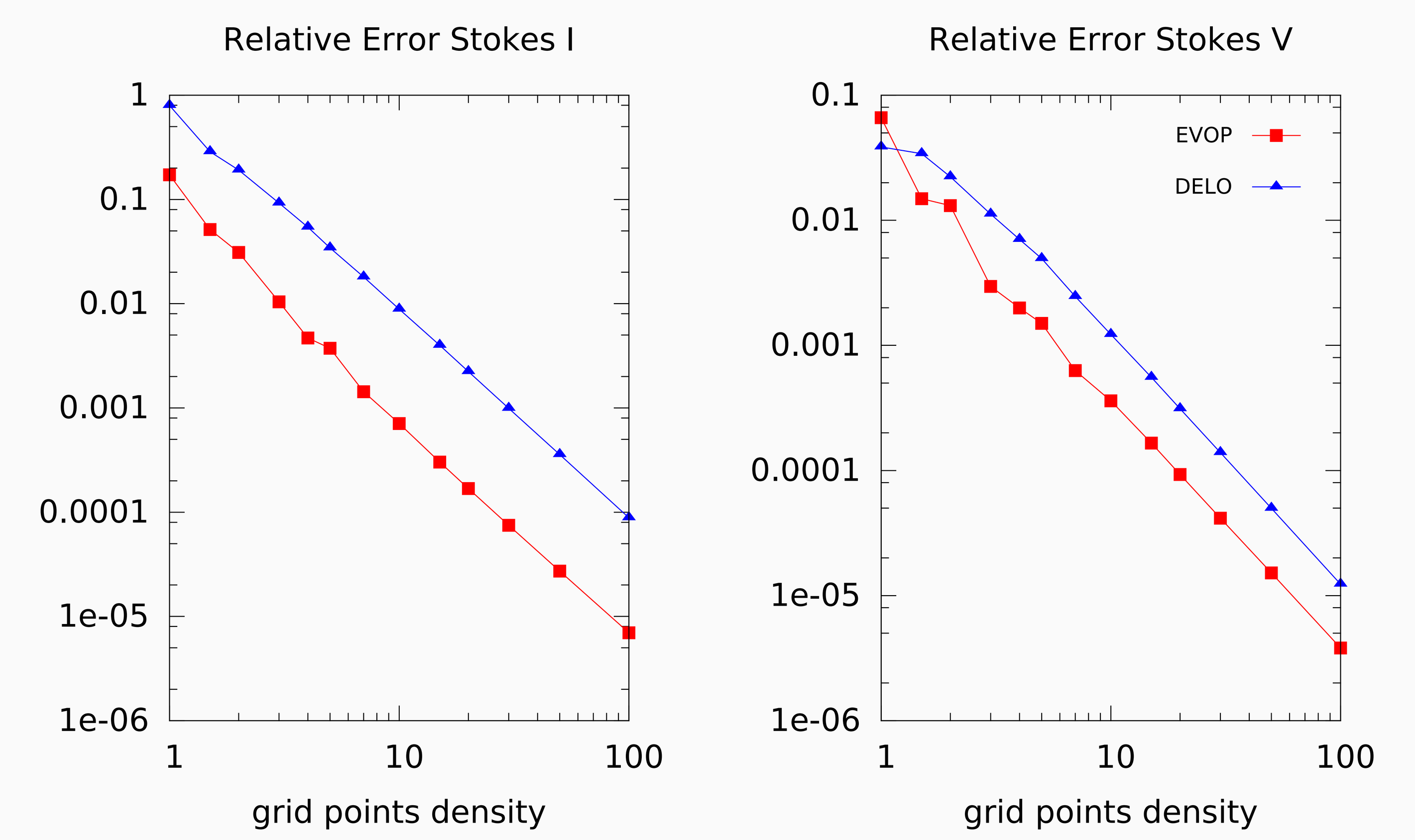
Atmosphere Parameters



Stokes Profiles



Convergence



Conclusions

The assessment of the evolution operator method with piecewise discontinuous approximation of the atmospheric parameters turns out positive. With respect to the more conventional method DELO, it leads to an order-of-magnitude improvement in the accuracy for the example considered here. Next, we plan to implement higher order interpolations for the emission, applying slope limiters in order to enforce monotonicity. Finally, the evolution operator method will be applied to more realistic and intermittent atmospheres from numerical simulations.

References

[1] E. Landi Degl'Innocenti and M. Landolfi. *Polarization in Spectral Lines*, volume 307 of *Astrophysics and Space Science Library*. Kluwer Academic Publishers, Dordrecht, August 2004.

[2] D. E. Rees, C. J. Durrant, and G. A. Murphy. Stokes profile analysis and vector magnetic fields. II - Formal numerical solutions of the Stokes transfer equations. *Astrophysical Journal*, 339:1093–1106, April 1989.